NAG Toolbox for MATLAB

g01er

1 Purpose

g01er returns the probability associated with the lower tail of the von Mises distribution between $-\pi$ and π through the function name.

2 Syntax

```
[result, ifail] = g01er(t, vk)
```

3 Description

The von Mises distribution is a symmetric distribution used in the analysis of circular data. The lower tail area of this distribution on the circle with mean direction $\mu_0 = 0$ and concentration parameter kappa, κ, can be written as

$$\Pr(\Theta \leq \theta : \kappa) = \frac{1}{2\pi I_0(\kappa)} \int_{-\pi}^{\theta} e^{\kappa \cos \Theta} \mathrm{d}\Theta,$$

where θ is reduced modulo 2π so that $-\pi \leq \theta < \pi$ and $\kappa \geq 0$. Note that if $\theta = \pi$ then g01er returns a probability of 1. For very small κ the distribution is almost the uniform distribution, whereas for $\kappa \to \infty$ all the probability is concentrated at one point.

The method of calculation for small κ involves backwards recursion through a series expansion in terms of modified Bessel functions, while for large κ an asymptotic Normal approximation is used.

In the case of small κ the series expansion of $\Pr(\Theta \leq \theta\colon \kappa)$ can be expressed as

$$\Pr(\Theta \leq \theta : \kappa) = \tfrac{1}{2} + \frac{\theta}{(2\pi)} + \frac{1}{\pi I_0(\kappa)} \sum_{n=1}^{\infty} n^{-1} I_n(\kappa) \sin n\theta,$$

where $I_n(\kappa)$ is the modified Bessel function. This series expansion can be represented as a nested expression of terms involving the modified Bessel function ratio R_n,

$$R_n(\kappa) = \frac{I_n(\kappa)}{I_{n-1}(\kappa)}, \qquad n = 1, 2, 3, \ldots,$$

which is calculated using backwards recursion.

For large values of κ (see Section 7) an asymptotic Normal approximation is used. The angle Θ is transformed to the nearly Normally distributed variate Z,

$$Z = b(\kappa) \sin \tfrac{\Theta}{2},$$

where

$$b(\kappa) = \frac{\sqrt{\frac{2}{\pi}} e^{\kappa}}{I_0(\kappa)}$$

and $b(\kappa)$ is computed from a continued fraction approximation. An approximation to order κ^{-4} of the asymptotic normalizing series for z is then used. Finally the Normal probability integral is evaluated.

For a more detailed analysis of the methods used see Hill 1977.

4 References

Hill G W 1977 Algorithm 518: Incomplete Bessel function I_0: The Von Mises distribution *ACM Trans. Math. Software* **3** 279–284

Mardia K V 1972 *Statistics of Directional Data* Academic Press

5 Parameters

5.1 Compulsory Input Parameters

1: **t – double scalar**

θ, the observed von Mises statistic measured in radians.

2: **vk – double scalar**

The concentration parameter κ, of the von Mises distribution.

Constraint: **vk** ≥ 0.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **vk** < 0 and g01er returns 0.

7 Accuracy

g01er uses one of two sets of constants depending on the value of ***machine precision***. One set gives an accuracy of six digits and uses the Normal approximation when **vk** ≥ 6.5, the other gives an accuracy of 12 digits and uses the Normal approximation when **vk** ≥ 50.

8 Further Comments

Using the series expansion for small κ the time taken by g01er increases linearly with κ; for larger κ, for which the asymptotic Normal approximation is used, the time taken is much less.

If angles outside the region $-\pi \leq \theta < \pi$ are used care has to be taken in evaluating the probability of being in a region $\theta_1 \leq \theta \leq \theta_2$ if the region contains an odd multiple of π, $(2n+1)\pi$. The value of $F(\theta_2;\kappa) - F(\theta_1;\kappa)$ will be negative and the correct probability should then be obtained by adding one to the value.

9 Example

```
t = 7;
vk = 0;
[result, ifail] = g01er(t, vk)
```

```
result =
    0.6141
ifail =
           0
```